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The Bethe Homework Problem for Hot Neutron Matter in the Static Fluctuation Approximation

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Abstract Nonrelativistic neutron matter is investigated in the static fluctuation approximation (SFA) within the framework of the Bethe homework problem. The idea is to introduce this new technique in the present context and compare it with other, better-known techniques. As a bonus, since the temperature enters SFA without any complexities, conceptual or otherwise, the thermodynamic properties of the system are calculated as functions of temperature.

Keywords Bethe homework problem · Hot neutron matter · Static fluctuation approximation

1 Introduction

The thermodynamic properties of dense and hot nuclear matter are a source of important theoretical predictions for the properties of heavy-ion collisions, neutron-star cooling, stellar collapse, and supernova explosions [1]. The characterization of nuclear matter properties, such as its phase diagram and equation of state, has been and remains the most eminent goal of nuclear physics. In particular, the evaluation of its thermodynamic properties starting from realistic models of the nucleon-nucleon (NN) interaction is still one of the challenging open problems in nuclear physics. The nuclear-matter phase diagram is dependent on density and temperature [2]. The critical temperature of the liquid-gas phase transition, based on the nonrelativistic theory near normal nuclear-matter density, is in the range of 14–20 MeV [3]. The equilibrium density of nuclear matter is 0.17 nucleons/fm^3, the interparticle spacing is 1.12 fm, the Fermi momentum is 1.36 fm^{-1}, and the equilibrium binding energy per nucleon is -15.7 MeV [4].

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Many microscopic techniques have been used over the years to study nuclear matter. These can be classified into two broad types: the first is perturbative, and the second type is variational.

In this work, nuclear matter is studied for the first time within the framework of the static fluctuation approximation (SFA) [5–7]. This is based on the replacement of the square of the local-field operator with its mean value. SFA has already been used to study various systems at low temperatures, such as neutral many-bosonic systems [5] and neutral many-fermionic systems [8]. Avoiding, as it does, Green's functions and the highly involved diagrammatic techniques, this is relatively simple compared with other conventional many-body approaches. Some many-body theories are somewhat inadequate at finite temperatures—unlike SFA. This approximation is, therefore, tested out here for *hot* nuclear matter as well.

This approximation will be examined here, *vis à vis* other techniques, within the Bethe homework problem [9] which was first suggested as a model potential for neutron matter of the Yukawa form, interacting via the purely repulsive part of the $^1S_0$ Reid soft-core NN interaction:

$$V(r)[\mathrm{MeV}] = 9263.1 \exp(-4.9r)/r. \tag{1}$$

The idea is to compare the various many-body theories in the field for a simple, albeit nontrivial, potential before a full-fledged NN potential is tackled.

The rest of the paper is organized as follows. In Sect. 2, the basic elements of SFA are introduced for hot nuclear matter. Next, in Sect. 3, the results of our calculations are presented and discussed. Finally, the paper is concluded, in Sect. 4, with general remarks.

2 Basic Elements of SFA for Hot Nuclear Matter

For a specific Hamiltonian $\hat{H}$, the Heisenberg representation of a creation operator $\hat{a}^+_{\vec{k}\lambda}(\tau)$ is given by

$$\hat{a}^+_{\vec{k}\lambda}(\tau) = \exp(\tau\hat{H})\hat{a}^+_{\vec{k}\tau}(0)\exp(-\tau\hat{H}), \tag{2}$$

where $\tau \equiv it$, t being the time. The equation of motion of the creation operator in this representation can be written in the form

$$\frac{d\hat{a}^+_{\vec{k}\lambda}}{d\tau} = [\hat{H}, \hat{a}^+_{\vec{k}\lambda}(\tau)]_-. \tag{3}$$

Similar equations apply to the annihilation operator $\hat{a}_{\vec{k}\lambda}(\tau)$.

Based on SFA [5], the total Hamiltonian describing a neutral many-fermionic system can be written as a linear combination of the local-field operator $\hat{E}_{\vec{k}\lambda}$ and the number-of-particles operator:

$$\hat{H} = \sum_{\vec{k}\lambda} \hat{E}_{\vec{k}\lambda}\hat{a}^+_{\vec{k}\lambda}\hat{a}_{\vec{k}\lambda}. \tag{4}$$

The indices $\vec{k}\lambda$ denote a complete set of quantum numbers. The creation and annihilation operators obey the well-known anticommutation relations for a Fermi system:

$$\{\hat{a}_{\vec{k}\lambda}, \hat{a}^+_{\vec{q}\lambda_1}\}_+ = \delta_{\vec{k}\vec{q}}\delta_{\lambda\lambda_1}; \qquad \{\hat{a}_{\vec{k}\lambda}, \hat{a}_{\vec{q}\lambda_1}\}_+ = 0. \tag{5}$$

Assuming that the local-field operator $\hat{E}_{\vec{k}\lambda}$ is hermitian and that it commutes with these operators, one can write

$$\frac{d\hat{a}^{+}_{\vec{k}\lambda}}{d\tau} = [\hat{H}, \hat{a}^{+}_{\vec{k}\lambda}(\tau)]_{-} = \hat{E}_{\vec{k}\lambda}\hat{a}^{+}_{\vec{k}\lambda}, \tag{6}$$

with a similar equation for $\hat{a}_{\vec{k}\lambda}(\tau)$.

The Hamiltonian for a neutral many-fermionic system with a central potential is given in the second-quantization representation by

$$\hat{H}_1 = \sum_{\vec{k}\lambda} \frac{\hbar^2 k^2}{2m} \hat{a}^{+}_{\vec{k}\lambda}\hat{a}_{\vec{k}\lambda} + \frac{1}{2\Omega} \sum_{\vec{k}\vec{q}\vec{p},\lambda\lambda_1} V(\vec{k})\hat{a}^{+}_{\vec{p}-\vec{k}\lambda}\hat{a}_{\vec{p}\lambda}\hat{a}^{+}_{\vec{q}\lambda_1}\hat{a}_{\vec{q}-\vec{k}\lambda_1}, \tag{7}$$

where m is the fermionic mass, Ω is the (normalization) volume of the system, the index $\vec{k}$ denotes the wavevector of the particle, and the index λ represents the spin of the particle; $V(\vec{k})$ is the Fourier transform of the pair potential defined as

$$V(\vec{k}) \equiv \int V(r)\exp(i\vec{k}.\vec{r})d\vec{r}. \tag{8}$$

The grand canonical Hamiltonian of this system is

$$\hat{H} = \sum_{\vec{k}\lambda} \varepsilon(k)\hat{a}^{+}_{\vec{k}\lambda}\hat{a}_{\vec{k}\lambda} + \frac{1}{2\Omega} \sum_{\vec{k}\vec{q}\vec{p},\lambda\lambda_1} V(k)\hat{a}^{+}_{\vec{p}-\vec{k}\lambda}\hat{a}_{\vec{p}\lambda}\hat{a}^{+}_{\vec{q}\lambda_1}\hat{a}_{\vec{q}-\vec{k}\lambda_1}, \tag{9}$$

where $\varepsilon(k) = \frac{\hbar^2 k^2}{2m} - \mu$, μ being the chemical potential. From (6) and (9), the local field operator can be calculated:

$$\hat{E}_{\vec{k}\lambda} = \{\hat{a}_{\vec{k}\lambda}, [\hat{H}, \hat{a}^{+}_{\vec{k}\lambda}]_{-}\}_{+} = \varepsilon(k) + \frac{1}{\Omega}\sum_{\vec{q}\lambda_1}[V(0) - V(\vec{q}-\vec{k})\delta_{\lambda_1\lambda}]\hat{a}^{+}_{\vec{q}\lambda_1}\hat{a}_{\vec{q}\lambda_1}. \tag{10}$$

From this equation, we find that the local-field operator is spin-symmetric: $\hat{E}_{\vec{k}\lambda_1} = \hat{E}_{\vec{k}\lambda_2} = \hat{E}_{\vec{k}}$. This is reasonable since the interaction potential $V(\vec{r}_1 - \vec{r}_2)$ is spin-independent; so is the particles distribution operator: $\hat{n}_{\vec{q}\lambda_1} = \hat{a}^{+}_{\vec{q}\lambda_1}\hat{a}_{\vec{q}\lambda_1} = \hat{a}^{+}_{\vec{q}\lambda_2}\hat{a}_{\vec{q}\lambda_2} = \hat{n}_{\vec{q}}$.

It follows that the local-field operator in the ground state is

$$\hat{E}_0 = \frac{1}{\Omega}\sum_{\vec{q}\lambda_1}[V(0) - V(\vec{q})\delta_{\lambda\lambda_1}]\hat{a}^{+}_{\vec{q}\lambda_1}\hat{a}_{\vec{q}\lambda_1}. \tag{11}$$

Further, the excitation local-field operator, as measured relative to $\hat{E}_0$, is

$$\hat{E}_{\vec{k}} \equiv \hat{E}_{\vec{k}} - \hat{E}_0 = \varepsilon(k) + \frac{1}{\Omega}\left[\sum_{\vec{q}} W(\vec{k},\vec{q})\hat{n}_{\vec{q}}\right], \tag{12}$$

where $W(\vec{k},\vec{q}) \equiv V(\vec{q}) - V(\vec{k}-\vec{q})$.

In SFA, $\hat{E}_{\vec{k}}$ is defined by its mean value $\langle\hat{E}_{\vec{k}}\rangle$ as well as the set of its fluctuations $\Delta\hat{E}_{\vec{k}}$. According to (11):

$$\langle\hat{E}_{\vec{k}}\rangle = \varepsilon(k) + \frac{1}{\Omega}\left[\sum_{\vec{q}} W(\vec{k},\vec{q})\langle\hat{n}_{\vec{q}}\rangle\right]. \tag{13}$$

The square of the quadratic fluctuation operator is replaced with its mean value:

$$(\Delta\hat{E}_{\vec{k}})^2 \cong \langle(\Delta\hat{E}_{\vec{k}})^2\rangle, \tag{14}$$

where $\Delta\hat{E}_{\vec{k}} \equiv \hat{E}_{\vec{k}} - \langle\hat{E}_{\vec{k}}\rangle$ is the corresponding deviation of the mean-field operator from its mean value. It follows that the local-field deviation operator has two symmetric eigenvalues:

$$(\Delta\hat{E}_{\vec{k}})^2 \cong \langle(\Delta\hat{E}_{\vec{k}})^2\rangle = \varphi_k^2, \tag{15}$$

which, in fact, defines a splitting of the operator $\Delta\hat{E}_{\vec{k}}$ produced by the quadratic fluctuations.

The long-range equations for neutral many-fermionic systems [8] are given by

$$\langle\hat{n}_{\vec{k}}\hat{A}\rangle = \eta_0(\vec{k})\langle\hat{A}\rangle + \eta_1(\vec{k})\langle\Delta\hat{E}_{\vec{k}}\hat{A}\rangle, \tag{16}$$

where $\hat{A}$ is any arbitrary operator that commutes with $\hat{a}^+_{\vec{k}\lambda}$, $\hat{a}_{\vec{k}\lambda}$ and $\hat{E}_{\vec{k}}$. Also,

$$\eta_0(\vec{k}) = \frac{1}{2}\left\{\frac{1}{\exp[\beta(\langle\hat{E}_{\vec{k}}\rangle + \varphi_k)]+1} + \frac{1}{\exp[\beta(\langle\hat{E}_{\vec{k}}\rangle - \varphi_k)]+1}\right\}; \tag{17a}$$

$$\eta_1(\vec{k}) = \frac{1}{2\varphi_k}\left\{\frac{1}{\exp[\beta(\langle\hat{E}_{\vec{k}}\rangle + \varphi_k)]+1} - \frac{1}{\exp[\beta(\langle\hat{E}_{\vec{k}}\rangle - \varphi_k)]+1}\right\}, \tag{17b}$$

where $\beta \equiv \frac{1}{k_B T}$, k_B being Boltzmann's constant and T the absolute temperature.

We can now find the closed system of nonlinear integral equations from the long-range equation (16). First, putting $\hat{A} = 1$ in this equation, we obtain the particle distribution, where the fluctuations are symmetric, $\langle\Delta\hat{E}_{\vec{k}}\rangle = 0$:

$$\langle\hat{n}_{\vec{k}}\rangle = \eta_0(\vec{k}). \tag{18}$$

It is more convenient to rewrite (16) in terms of the fluctuations of the occupation-number operator, this being defined as

$$\hat{n}_{\vec{k}} \equiv \langle\hat{n}_{\vec{k}}\rangle - \Delta\hat{n}_{\vec{k}}, \tag{19}$$

where we have taken the local-field operator as $\hat{E}_{\vec{k}} = \langle\hat{E}_{\vec{k}}\rangle + \Delta\hat{E}_{\vec{k}}$. Therefore, the occupation-number operator must be defined as in (19); this is because there is a relative sign between the fluctuations in the number-of-particles operator and the fluctuations in the local-field operator. In terms of the fluctuations of the occupation-number operator, the long-range equation is

$$\langle\Delta\hat{n}_{\vec{k}}\hat{A}\rangle = -\eta_1(\vec{k})\langle\Delta\hat{E}_{\vec{k}}\hat{A}\rangle. \tag{20}$$

Putting $\hat{A} = \Delta\hat{n}_{\vec{q}}$ in this equation, where $\vec{q} \neq \vec{k}$, we have the pair correlation function $\langle\Delta\hat{n}_{\vec{k}}\Delta\hat{n}_{\vec{q}}\rangle_c$, the index c denoting the true correlations $\vec{q} \neq \vec{k}$:

$$\langle\Delta\hat{n}_{\vec{k}}\Delta\hat{n}_{\vec{q}}\rangle_c = -\eta_1(\vec{k})\langle\Delta\hat{E}_{\vec{k}}\Delta\hat{n}_{\vec{q}}\rangle_c = -\frac{\eta_1(\vec{k})}{\Omega}\sum_{\vec{p}} W(\vec{k},\vec{p})\langle\Delta\hat{n}_{\vec{p}}\Delta\hat{n}_{\vec{q}}\rangle. \tag{21}$$

The correlation function $\langle\Delta\hat{n}_{\vec{p}}\Delta\hat{n}_{\vec{q}}\rangle$ can be written as

$$\langle\Delta\hat{n}_{\vec{p}}\Delta\hat{n}_{\vec{q}}\rangle = \langle(\Delta\hat{n}_{\vec{q}})^2\rangle\delta_{\vec{q}\vec{p}} + \langle\Delta\hat{n}_{\vec{p}}\Delta\hat{n}_{\vec{q}}\rangle_c. \tag{22}$$

The value $\langle(\Delta\hat{n}_{\vec{q}})^2\rangle$ is determined as

$$\langle(\Delta\hat{n}_{\vec{q}})^2\rangle = \langle\hat{n}_{\vec{q}}^2\rangle - \langle\hat{n}_{\vec{q}}\rangle^2. \tag{23}$$

For fermionic systems $\hat{n}_{\vec{q}}^2 = \hat{n}_{\vec{q}}$; so that (23) becomes

$$\langle(\Delta\hat{n}_{\vec{q}})^2\rangle = \langle\hat{n}_{\vec{q}}\rangle(1 - \langle\hat{n}_{\vec{q}}\rangle). \tag{24}$$

To close our system of nonlinear integral equation, we have, putting $\hat{A} = \Delta\hat{E}_{\vec{k}}$ in (20),

$$-\eta_1(\vec{k})\varphi_k^2 = \frac{1}{\Omega}\sum_{\vec{p}} W(\vec{k},\vec{p})\langle\Delta\hat{n}_{\vec{p}}\Delta\hat{n}_{\vec{k}}\rangle. \tag{25}$$

The set of nonlinear integral equations (13), (18), (21), (24) and (25) can be solved numerically to calculate the thermodynamic properties of the system.

In the thermodynamic limit, the summation in (13), (21) and (25) can be changed to integration. After integrating over the solid angle and multiplying by two for equivalent spins (spin-up and spin-down), these equations reduce to

$$\langle\hat{E}_k\rangle = \varepsilon(k) + \frac{1}{2\pi^2}\int_0^\infty W(k,p)\langle\hat{n}_p\rangle p^2 dp; \tag{26}$$

$$\langle\Delta\hat{n}_k\Delta\hat{n}_q\rangle_c = -\frac{\eta_1(k)}{2\pi^2}\int_0^\infty W(k,p)\langle\Delta\hat{n}_p\Delta\hat{n}_q\rangle p^2 dp; \tag{27}$$

$$-\eta_1(k)\varphi_k^2 = \frac{1}{2\pi^2}\int_0^\infty W(k,p)\langle\Delta\hat{n}_p\Delta\hat{n}_k\rangle_c p^2 dp; \tag{28}$$

$W(k,p) \equiv V(p,0) - V(k,p)$, $V(k,p)$ being the Fourier-Bessel transform of the potential:

$$V(k,p) = 4\pi\int_0^\infty j_L(kr)V(r)j_{L'}(pr)r^2 dr, \tag{29}$$

where $j_L(kr)$and $j_{L'}(pr)$ are spherical Bessel functions.

The integrals in (26)–(28) are calculated using Gaussian quadrature [10, 11]. Our set of nonlinear integral equations can then be solved numerically by an iteration method for the Bethe homework problem. We use here a natural system of units such that $\hbar = m = k_B = 1$, where m is the nucleon mass, the conversion unit being $\hbar^2/m = 41.467$ MeV fm^2.

The thermodynamic properties of the system [5] can readily be obtained from the grand partition function Q:

$$\ln Q = \sum_{\vec{p}\lambda} q_0(p), \tag{30}$$

where

$$q_0(p) \equiv \frac{1}{2}[\ln\{1 + \exp[-\beta(\langle\hat{E}_p\rangle + \varphi_p)]\} + \ln\{1 + \exp[-\beta(\langle\hat{E}_p\rangle - \varphi_p)]\}]. \tag{31}$$

The grand mean energy $\langle\hat{H}\rangle$ is defined as

$$U = \langle\hat{H}\rangle = -\frac{\partial\ln Q}{\partial\beta} = \sum_{\vec{p}\lambda}[\langle\hat{E}_p\rangle\langle\hat{n}_p\rangle + \eta_1(p)\varphi_p^2]. \tag{32}$$

From the grand partition function and the grand mean energy, it is simple to evaluate the other thermodynamic properties:

The usual definition of the pressure is given by

$$P = k_B T \frac{\ln Q}{\Omega}. \tag{33}$$

The entropy of the system can be evaluated from the first law of thermodynamics in terms of the grand mean internal energy U and the pressure:

$$S = \frac{1}{T}(U + P\Omega). \tag{34}$$

Finally, the heat capacity of the system at constant volume is

$$C_v = \left(\frac{\partial U}{\partial T}\right)_{\Omega}. \tag{35}$$

3 Results and Discussion

The thermodynamic properties of hot neutron matter were studied here within SFA for the Bethe-homework-problem potential in the temperature range 5–50 MeV and density range 0.05–0.45 fm^{-3}.

Our ground-state energy per nucleon for neutron matter at different densities is first compared with previous results. Table 1 shows this comparison. There is good agreement between our calculations and Jastrow-Monte Carlo (J-MC) calculations [9] as well as with Fermi hypernetted-chain variational calculations (FHNC) [12]. In addition, our results are consistent with the Padé extrapolation techniques (PET) [13]. So are they with the results of Chakravarty et al. [14], who performed variational calculations for both liquid and solid phases of neutron matter, using Boltzmann statistics for simplification purposes; see, in particular, their Fig. 3. Shen and Woo [15] compared the results for the energy density of a neutron liquid, as calculated by different techniques using the Bethe 'homework' potential. They found good agreement between their results and other workers', as shown in their Fig. 2. Our results lie in the same range. So do the results of the constrained variational method at high density [16], where the neutrons were again treated as Boltzmann particles.

Figures 1–4 show the thermodynamic properties of our system as functions of temperature at different values of density. In these and all other figures, the lines joining the plotted points have been added just to guide the eye. It is clear that the mean internal energy per unit volume U/Ω, the pressure P, the entropy per unit volume S/Ω, and the heat capacity per unit volume C_v/Ω monotonically increase with increasing temperature. However, there

Table 1 The SFA ground-state energy per nucleon (in MeV) of the "Bethe-homework problem-potential" for neutron matter, compared with Jastrow-Monte Carlo (J-MC) calculations, Fermi hypernetted-chain variational calculations (FHNC), and Padé extrapolation techniques (PET)

ρ (fm^{-3})	J-MC [9]	FHNC [12]	PET [13]	This work
0.17	89.6 ± 0.7	90	85.93	81.47
0.30	174.9 ± 0.7	177	162.95	157.33

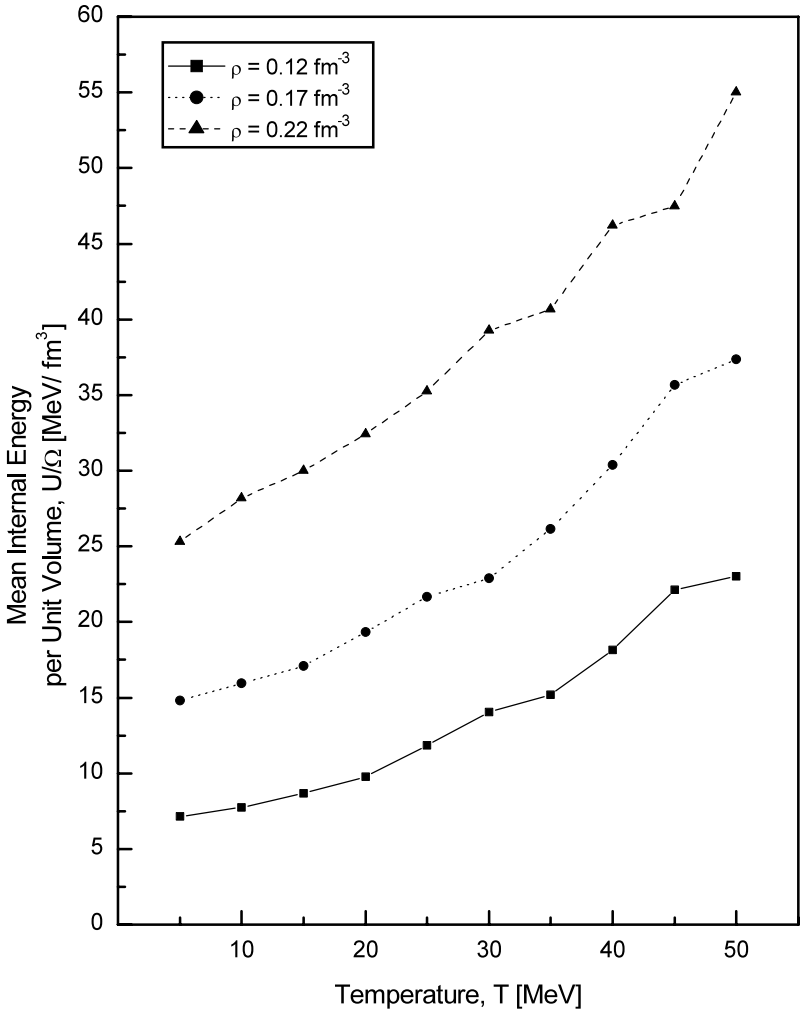

Fig. 1 The mean internal energy per unit volume U/Ω of neutron matter as a function of temperature T at three different values of density ρ using the Bethe-homework potential. In this and all other figures, the lines joining the plotted points have been added just to guide the eye

is an onset of fluctuations above certain values of (ρ, T). This may indicate a restriction on the range of validity of the Bethe potential at relatively high density and temperature.

From our results we find that all the thermodynamic properties increase with density except C_v/Ω. Figure 5 represents C_v/Ω as a function of density at different temperatures. From this figure, C_v/Ω has a peak around $\rho = 0.17$ fm^{-3} at $T = 20$ MeV and $T = 35$ MeV; this is just the equilibrium density of nuclear matter. On the other hand, C_v/Ω has a maximum density around $\rho = 0.25$ fm^{-3} at $T = 50$ MeV. From these results we conclude that the equilibrium density is temperature-independent at least up to $T = 35$ MeV. This behavior of C_v/Ω means that the system becomes disordered around the equilibrium density.

The chemical potential μ, shown in Fig. 6, decreases with increasing temperature. The general behavior of the chemical potential is consistent with that of an ideal Fermi system.

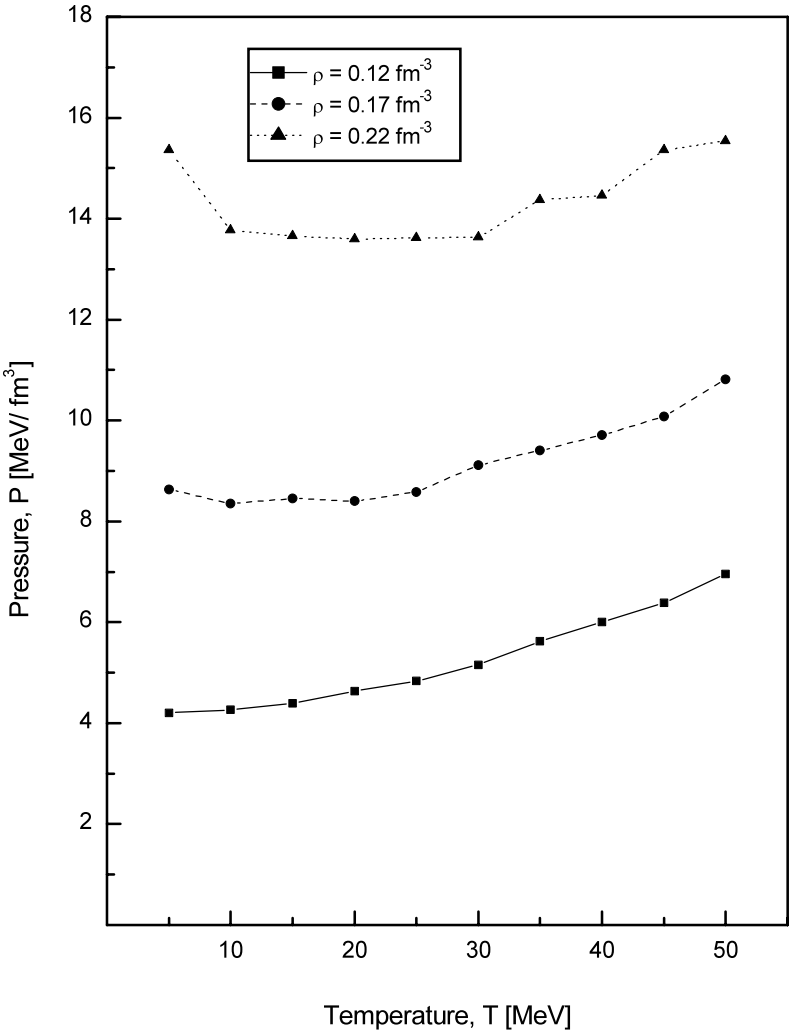

Fig. 2 The pressure P of neutron matter as a function of temperature T at three different values of density ρ using the Bethe-homework potential

It is clear from Fig. 6 that the transition temperature from the quantum-mechanical regime to the classical regime is density-dependent.

Clearly, there is good agreement between our SFA results and those obtained by other well-known techniques—including the (lowest-order) constrained variational, Monte Carlo, correlated-basis-function, and hypernetted chain methods. It will be interesting to see whether this agreement would extend to finite-temperature properties. At present, no such results are available for those techniques. Perhaps a new temperature-dependent homework problem is called for.

Neutron matter was studied at both zero and finite temperatures by several groups using different techniques [17–23]. Our results are, by and large, consistent with those works. The Fermi hypernetted chain (FHNC) method was used [17] for two- and three-nucleon

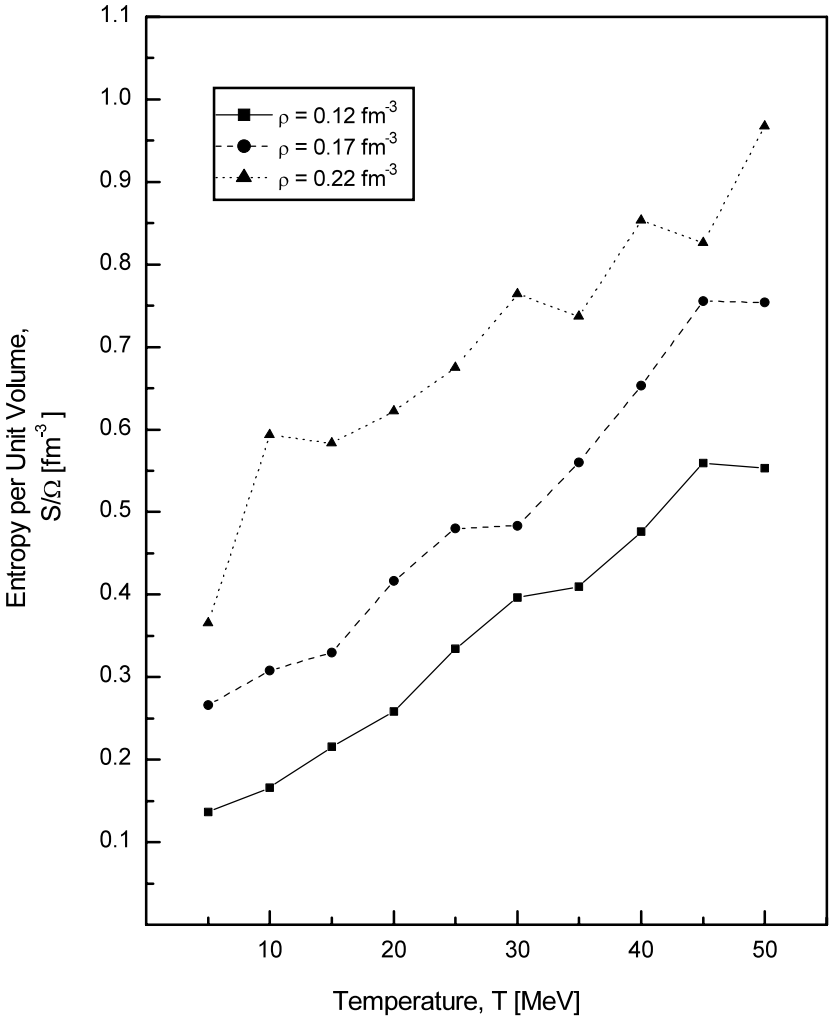

Fig. 3 The entropy per unit volume S/Ω of neutron matter as a function of temperature T at three different values of density ρ using the Bethe-homework potential

interactions. The pressure is calculated as a function of density in the temperature range $T \leq 20$ MeV. It turns out that the pressure at any density increases with temperature. This behavior is consistent with our results. Also, the entropy per nucleon was calculated. In our work, we calculated the entropy per unit volume. To compare, a suitable conversion factor was used. We find that both results are consistent with each other, although in general our results lie above those results. Likewise, the FHNC method was later used [18] to calculate the energy and entropy per nucleon at different temperatures as functions of density. Again, our results are greater than those results; however, there is an overall agreement in behavior between both sets of results.

In a different track, hot nuclear matter was studied in relativistic heavy-ion collisions [19], including pion production. The total and nucleon entropies were calculated. The thermal pion effect appeared at temperatures >40 MeV. The temperature-dependence of our

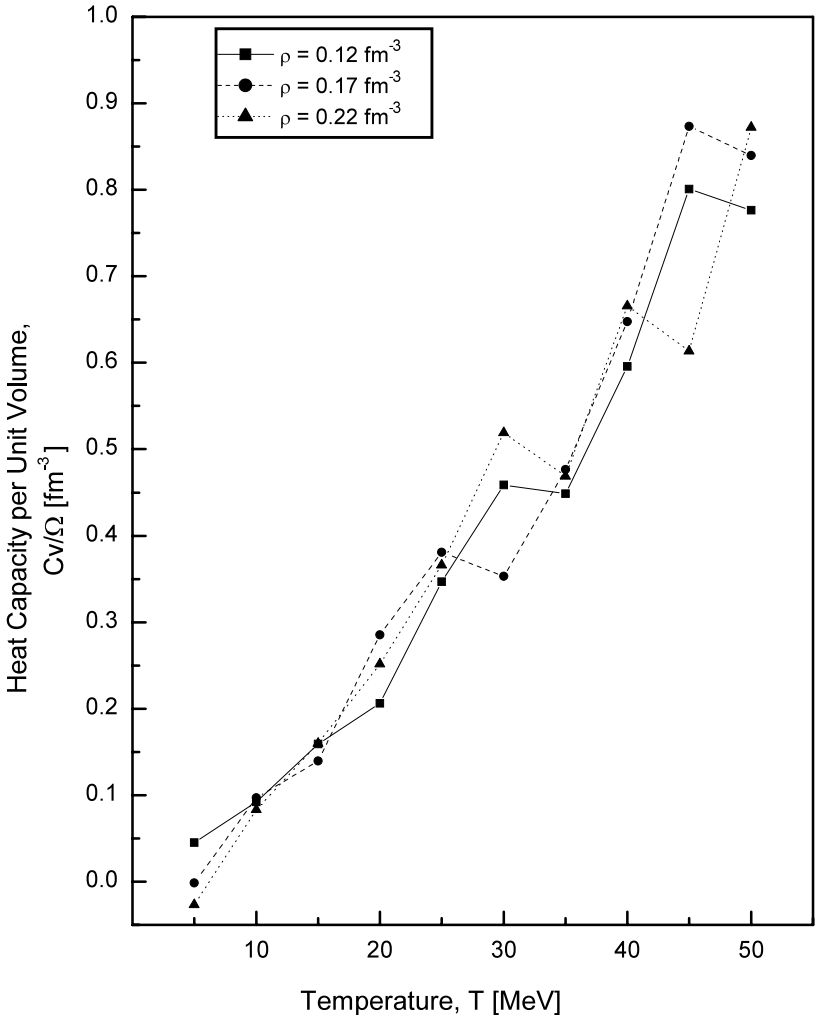

Fig. 4 The heat capacity per unit volume C_v/Ω of neutron matter as a function of temperature T at three different values of density ρ using the Bethe-homework potential

results is in good agreement with those results for nucleon entropy. In our work, the entropy per unit volume, at $T = 20$ MeV and $\rho = 0.12$ fm^{-3}, $\sim$0.25 fm^{-3}; and, at the same T and $\rho = 0.17$ fm^{-3}, $\sim$0.4 fm^{-3}. In their work, the entropy per unit volume (after converting from entropy per nucleon) at the same T and $\rho = 0.1589$ fm^{-3} $\sim$0.3 fm^{-3}. This simple comparison indicates that at relatively low temperatures ($T < 40$ MeV), the central potential is presumably sufficient to describe the interaction between nucleons.

In addition, the lowest-order constrained variational (LOCV) method was used [20, 21] to investigate both neutron and nuclear matter. The pressure was studied [20] at different values of temperature as a function of density, using the modified Reid potential. These authors find that the pressure increases with increasing density and temperature, which is in agreement with our results. The effect of three-body cluster energy on the thermodynamic properties was also studied [21]. Even with this additional contribution, there is still good

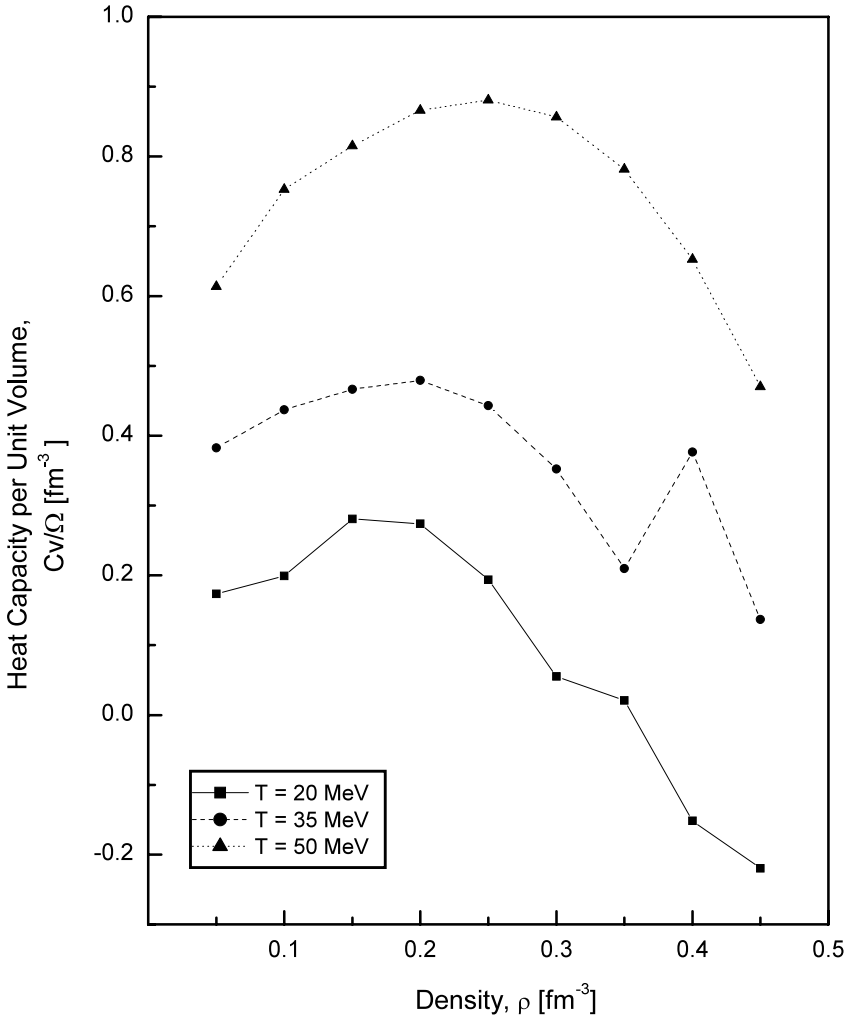

Fig. 5 The heat capacity per unit volume C_v/Ω of neutron matter as a function of density ρ at three different values of temperature T using the Bethe-homework potential

agreement between our results and their results. As an example, the entropy per unit volume at $T = 10$ MeV and $\rho = 0.12$ fm^{-3} $\sim$0.1 fm^{-3} in our work, and $\sim$0.16 fm^{-3} in their work.

Finally, the virial equation of state of low-density neutron matter was investigated [22] as well as the equation of state for neutron matter using the renormalization-group approach [23]. The results of theses two works are consistent with each other as well as with our work.

4 Conclusions

In this paper, we have studied the thermodynamic properties of nonrelativistic neutron matter in the SFA using the Bethe-homework-problem potential.

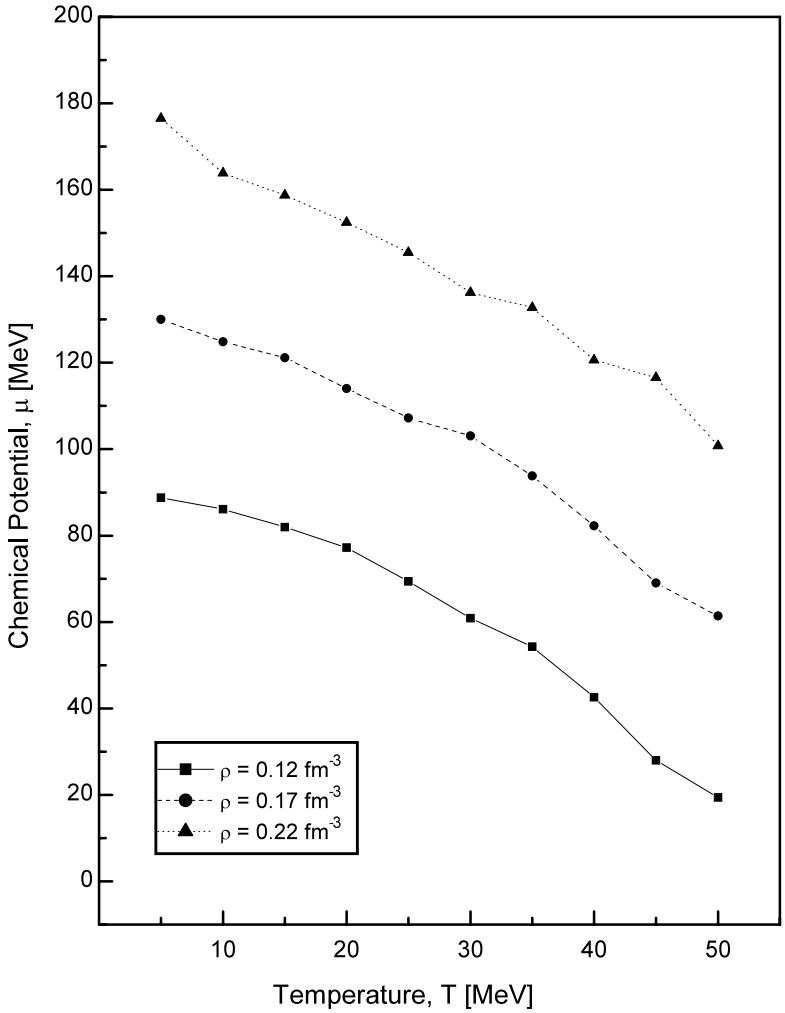

Fig. 6 The chemical potential μ of neutron matter as a function of temperature T at three different values of density ρ using the Bethe-homework potential

The temperature enters quite naturally in SFA without any complications—conceptual or calculational.

The SFA ground-state energy per nucleon is comparable with the best techniques available. This has encouraged us to go farther afield within SFA to a full-fledged calculation with a realistic NN potential, e.g., Reid93 potential [24].

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